Steady-State Light-Induced Forces for Atom Lithography

A. V. Bezverbny1,*, O. N. Prudnikov2, A. V. Taichenachev2, A. M. Tumaikin3, and V. I. Yudin3

1 Maritime State University, Vladivostok, 690059 Russia
2 Novosibirsk State University, Pirogova 2, Novosibirsk, 630090 Russia
3 Institute of Laser Physics, Siberian Division, Russian Academy of Sciences, pr. Akademika Lavrent’eva 13/3, Novosibirsk, 630090 Russia

*e-mail: alexb@mail.vntc.ru
Received December 27, 2004

Abstract—The general properties of light-induced forces arising in dissipative light masks are considered. The localization sites of atoms depending on detuning and ellipticity are determined for an arbitrary configuration of monochromatic light beams resonant with two types of transitions: \( J \rightarrow J \), where \( J \) is a half-integer, and \( J \rightarrow J + 1 \), where \( J \) is arbitrary.

INTRODUCTION

Atom lithography with the direct deposition technique is one of the most useful applications of light-induced forces for producing nanostructures. Here, a planar configuration of light beams, acting like a mask on a previously collimated atomic beam, generates thickness-modulated structures on a suitable substrate (Fig. 1). Current results and potential applications in connection with the formation of such nanostructures are the subjects of [1, 2].

Two different regimes of the light-mask operation have been considered so far. In the first case [1, 2], it is assumed that atoms interact with a far-off-resonance light field in a coherent manner. The effect consists in the focusing of an atomic beam in conservative adiabatic potentials originating from the spatially modulated energy light shifts. The other dissipative regime for light masks was presented recently in [3, 4]. It combines focusing with laser cooling, resulting in a smaller width and higher contrast of the spatial atomic structures. In the framework of this method, there are conservative, dissipative, and random contributions to the total light force on the atom.

We consider the influence of spontaneous emission and optical pumping on the spatial structure of the light force on an atom at rest: \( \mathbf{F}_0(\mathbf{r}) \). In the regime of a dissipative light mask, the force and its fluctuations should be calculated with an accounting for optical pumping. In the steady-state regime, when the atom–light interaction time exceeds the optical pumping time, we obtain compact analytical expressions for \( \mathbf{F}_0(\mathbf{r}) \) in a monochromatic field of arbitrary spatial structure and arbitrary intensity. The spatial structure of \( \mathbf{F}_0(\mathbf{r}) \) and its dependence on field invariants are studied in 1D as well as in 2D cases. These results can be applied to both the conservative and dissipative light masks.

1. STATEMENT OF THE PROBLEM

We consider a beam of atoms with the total angular momenta \( J_g \) in the ground state and \( J_e \) in the excited state. The atomic transition \( J_g \rightarrow J_e \) is resonantly driven by a monochromatic field formed by \( s \) coherent light beams with the wave vectors \( \mathbf{k}_n \) lying in the plane orthogonal to the atomic beam direction:

\[
\mathbf{E}(\mathbf{r}, t) = e^{-i\omega t} \mathbf{E}(\mathbf{r}) + \text{c.c.},
\]

\[
\mathbf{E}(\mathbf{r}) = \sum_{n=1}^{s} e^{i\mathbf{k}_n \cdot \mathbf{r}} \mathbf{E}_n = e^{i\Phi} \mathbf{E},
\]

where the unit polarization vector \( \mathbf{e} \) \((e^* \cdot \mathbf{e}) = 1 \) and \( \text{Im}(\mathbf{e} \cdot \mathbf{e}) = 0 \), the phase \( \Phi \), and the real amplitude \( \mathbf{E} \) of the total field are functions of the spatial coordinates \( \mathbf{r} \). Hereinafter we will use the following scalar field invariants: \( \mathcal{E}^2 = \mathbf{E} \cdot \mathbf{E}^* \) and \( \mathcal{F} = \mathbf{E} \cdot \mathbf{E} = e^{2i\Phi}(\mathbf{e} \cdot \mathbf{e}) \mathcal{E}^2 \). The first of them is proportional to the local energy density (intensity), and the other \( \mathcal{F} \) contains information on the local phase and polarization of the field. In particular, the degree of linear polarization can be expressed as \( l = (\mathbf{e} \cdot \mathbf{e}) = \sqrt{\mathcal{F} \cdot \mathcal{F} / \mathcal{E}^2} \).

For atoms with a degenerate ground state, the rate of optical pumping can be estimated as \( \Gamma_{\text{op}} = \gamma\pi \), where \( \gamma \) is the excited-state radiative relaxation rate and \( \pi \) is the total excited-state population. The rate of the dynamics of the atomic distribution on the translational degrees of freedom is of the order of \( (\hbar k/\Delta p) \Gamma_{\text{scat}} \), where \( \Gamma_{\text{scat}} \) is the rate of the scattering of photons by the atom. As is well known [5, 6], under these conditions one can apply an approximate quasi-classical treatment to the translational degrees of freedom. In such a way, the classical notion of the light-induced force on an atom emerges. For an atomic beam precooled in the transverse direction to the Doppler or sub-Doppler temperatures, we use the slow-atom approximation [6], which is valid when the atom displacement during the optical pump-
ing time is much less than the field spatial period \( \nu(\Gamma_{\text{op}})^{-1} \ll \lambda \). In the framework of this approximation, the light force is expanded in a series

\[
F(r, \nu) = F_0(r) + \hat{X}(r) \nu + \ldots,
\]

where \( F_0(r) \) is the force acting on the atom at rest in \( r \). It is responsible for the focusing and localization of atoms. The symmetric part of the Cartesian tensor \( \hat{X} \) governs the process of linear momentum dissipation due to interaction with atoms. The symmetric part of the Cartesian tensor \( \hat{X} \) can be decomposed in the field spatial gradients [6]:

\[
F_0 = \hbar \gamma \sum_{i=1}^{4} F_i \mathbf{g}_i,
\]

\[
\hat{X} = \hbar \left[ \sum_{i,j=1}^{4} X_{ij} \mathbf{g}_i \otimes \mathbf{g}_j + \sum_{l,m=5}^{6} X_{lm} \mathbf{g}_l \otimes \mathbf{g}_m \right],
\]

where \( \mathbf{g}_i \otimes \mathbf{g}_j \) denotes the direct product of the vectors \( \mathbf{g}_i \) and \( \mathbf{g}_j \). Here, explicit expressions of field gradients \( \mathbf{g}_i \) are defined in [6, 7]: \( \mathbf{g}_1 \) is related to the intensity gradient; \( \mathbf{g}_2 \) is the gradient of the total phase; \( \mathbf{g}_3 \) is connected with the gradient of the ellipticity; \( \mathbf{g}_4, \mathbf{g}_5, \) and \( \mathbf{g}_6 \) are gradients of angles determining the orientation of the polarization ellipse with respect to a given coordinate frame.

We consider the steady-state regime in which the atom–light interaction time \( \tau \) is much larger than the optical pumping time:

\[
\Gamma_{\text{op}} \tau \gg 1.
\]

2. LIGHT FORCE ON AN ATOM AT REST

According to the results presented in [8] for the two transitions \( J \rightarrow J + 1 \) and \( J \rightarrow J \), where \( J \) is a half-integer, the coefficients \( F_i \) are written as:

\[
F_1 = -2 \tilde{\delta} \pi_e, \quad F_2 = \pi_e, \quad F_3 = -\frac{\tilde{\delta} \pi_e (1 - A / \alpha_1)}{l}, \quad F_4 = -\pi_e A / \alpha_1 \sqrt{1 - l^2}, \quad \pi_e = \frac{S \alpha_1}{\alpha_0 + 2 S \alpha_1},
\]

where \( \pi_e \) is the total excited-state population, \( \tilde{\delta} = \delta / \gamma \) is the detuning in \( \gamma \) units, and \( S = 1 / (2 + 8 \tilde{\delta}^2) (I / I_{\text{sat}}) \) is the local saturation parameter defined in such a way that it is equal to one-half at zero detuning and the intensity \( I \) is equal to the saturation intensity \( I_{\text{sat}} = 2 \pi^2 \hbar c / (3 \lambda^3) \). The other coefficients \( \alpha_0, \alpha_1, \) and \( A \) depend on the local field ellipticity only. In the case of \( J \rightarrow J + 1 \) transitions, the following is true:

\[
\alpha_0 = \frac{1}{(2J + 1)(4J + 1)l} \sum_{p = 0}^{2J} C_p P_p (I^{-1});
\]

\[
\alpha_1 = P_{2J+1}(I^{-1}), \quad A = \frac{P_{2J+1}(I^{-1})}{(2J + 1)l},
\]

\[
C_p = (2p + 1)(2J - p)!(2J + p + 1)!
\]

where \( P_p(x) \) are Legendre polynomials, \( \kappa = 0 \) for integer \( J \), and \( \kappa = 1 \) for half-integer \( J \). For the other type of \( J \rightarrow J \) (a half-integer) transitions, they read:

\[
\alpha_0 = \frac{4J(2J+1)}{l} \sum_{p=1}^{2J} C_p P_p (I^{-1});
\]

\[
\alpha_1 = 2J + 1, \quad A_0 = 0,
\]

\[
C_p = (2p + 1) \left[ \frac{(p-1)!!}{p!!} \right] (2J + p)!(2J - p)!! (2J + p + 1)!!
\]
The force \( \mathbf{F}_0 \) is a sum of the scattering force and the dipole force:

\[
\mathbf{F}_{\text{scatt}} = \hbar \gamma (\mathbf{F}_2 \mathbf{g}_2 + \mathbf{F}_4 \mathbf{g}_4); \quad \mathbf{F}_{\text{dip}} = \hbar \gamma (\mathbf{F}_1 \mathbf{g}_1 + \mathbf{F}_3 \mathbf{g}_3).
\]

The scattering force originates from the stimulated absorption and subsequent spontaneous emission of photons. It has an absorption-like even dependence on the detuning. The dipole force is caused by the coherent rescattering of photons between different laser beams. Since it has a dispersive odd dependence on the detuning, its contribution into the force \( \mathbf{F}_0 \) dominates at large detunings \( |\delta| \geq 10 \). Thus, the dipole force is of major interest for atom lithography applications.

3. DIPOLE FORCE

The spatial dependence of the dipole force has the form

\[
\mathbf{F}_{\text{dip}} = -\hbar \delta \pi_e(r) \nabla \Psi(r) \tag{6}
\]

of the product of the total excited-state population \( \pi_e(r) \) and the gradient of the dimensionless scalar function \( \Psi(r) \). In the general case, the dipole force is not potential. Nevertheless, the function \( \Psi \) plays the role of a potential in many aspects. For example, all equilibrium points, where the dipole force vanishes, correspond to minima or maxima of \( \Psi \). Zeros of the total excited-state population \( \pi_e(r) \) do not give additional equilibrium points, as is seen from the explicit expressions for \( \Psi \). Moreover, at positive (negative) detuning, the dipole force attracts atoms to the minima (maxima) of the function \( \Psi \). Eventually, in the high-saturation limit when \( S \alpha_1/\alpha_2 \gg 1, \pi_e = 1/2 \) and the dipole force becomes a potential with a potential proportional to \( \Psi \).

3.1. Transitions \( J \rightarrow J \) with \( J \) being a Half-Integer

For these transitions, the function \( \Psi(r) \) in (6) has the form

\[
\Psi = \ln(|\mathcal{F}|). \tag{7}
\]

The dipole force \( \mathbf{F}_{\text{dip}} \) is not saturated in the vicinity of points with circular polarization \( (l = 0) \), where it can be approximated as \( \mathbf{F}_{\text{dip}} \approx -\nabla U \) in terms of the effective potential:

\[
U_{\text{eff,}1} = \hbar \delta S \frac{1}{(2J)(2J + 2)} \left[ \frac{(2J)!!}{(2J - 1)!!} \right]^2 l^{2J + 1}. \tag{8}
\]

The absence of saturation in the case of pure circular polarization (say \( \sigma_+ \)) of the total field is physically obvious, because all atoms are optically pumped into the dark state \( |J_e, m_e = J_e \rangle \). Correspodingly, in these points the light-induced force \( \mathbf{F}_0 \) vanishes and the potential \( U_{\text{eff,}1} \) has a minimum at positive detunings for arbitrary field configurations.

When \( l \neq 0 \), the dipole force \( \mathbf{F}_{\text{dip}} \) has the following properties. In the high-saturation limit \( S \alpha_1/\alpha_2 \gg 1 \), it is completely determined by the optical potential

\[
U_{\text{eff,}2} = \frac{\hbar \delta}{2} \ln(|\mathcal{F}|). \tag{9}
\]

At moderate saturation \( S \leq 1 \), the force is not purely potential. One can see that the equilibrium points, where \( \mathbf{F}_{\text{dip}} = 0 \), are governed not only by the spatial distribution of the light intensity \( l \) but also by the extrema of the function \( |\mathcal{F}| = |\mathbf{E} \cdot \mathbf{E}| \). Moreover, the nodes of the total field \( (\mathbf{E} = 0) \) are the localization points at positive detunings. Therefore, the regime of atomic lithography with \( \delta > 0 \) exists, wherein atoms are attracted to the points of circular polarization and to the field nodes in an arbitrary light mask. Additional localization sites can arise from the local minima of the function \( |\mathcal{F}| \). For negative detunings the atomic pattern is determined by the maxima of the function \( |\mathcal{F}| \).

3.2. Transitions \( J \rightarrow J + 1 \)

Here, the scalar function \( \Psi \) is given by

\[
\Psi = \frac{1}{2J + 1} \ln((\mathcal{E} l)^{(2J + 1)} P_{2J + 1}((l^{-1}))). \tag{9}
\]

The function \( l^{2J + 1} P_{2J + 1}(l^{-1}) \) is even in \( l \). It is maximal at circular polarization \( (l = 0) \) and decreases monotonically with an increase of \( |l| \). In the high-saturation limit \( (S \gg 1) \), the force \( \mathbf{F}_{\text{dip}} \) corresponds to the potential

\[
U_{\text{eff,}3} = \frac{\hbar \delta}{2} \Psi. \tag{9}
\]

In the general case, the dipole force contains a vertical component in addition to the potential one. However, its influence on the spatial layout of preferable sites of localization can be neglected. For this type of transition, the localization sites are determined by both the distribution of the light intensity \( l \) and the spatial dependence of the function \( |\mathcal{F}| \). In the general case, these points do not coincide either with points of circular (linear) polarization or with nodes (antinodes) of the field. In other words, they are governed by the spatial structures of the three functions \( \mathcal{E}(r), l(r), \Psi(r) \) simultaneously.

4. 1D FIELD CONFIGURATIONS

We consider symmetrical one-dimensional field configurations wherein the light force corresponds to a periodic potential, which are most interesting in the context of atom lithography. All such configurations are formed by two counterpropagating (along the \( z \) axis) light waves with equal amplitudes \( E \). They can be unambiguously classified by three parameters: the angle \( \theta \) between the major semiaxes of the polarization...
ellipses of the waves and two ellipticity angles \( \epsilon_1 \) and \( \epsilon_2 \). The ellipticity angle is defined in such a way that \( |\tan \epsilon| \) is equal to the ratio of the minor ellipse semiaxis to the major one, and the sign of \( \epsilon \) governs the helicity.

We consider \( \epsilon_1 - \theta - \epsilon_2 \) field configurations at some specific values of the parameters. We always have two simple rules. First, all equilibrium points, where the dipole force vanishes, correspond either to maxima or minima of the function \( \Psi \). Second, at large \( \delta > 0 \) (\( \delta < 0 \)), atoms are attracted to minima (maxima) of the function \( \Psi \).

We then proceed with more complicated field configurations, where two and four gradients \( \mathbf{g} \) differ from zero simultaneously. Among all \( \epsilon_1 - \theta - \epsilon_2 \) field configurations, we can distinguish three symmetrical classes for which the optical potential is periodic or, alternatively, for which the light force on an atom at rest vanishes under spatial averaging over the field period \( \lambda \). The first of these classes corresponds to \( \epsilon_1 = -\epsilon_2 = \epsilon \), and it will be referred to as \( \epsilon - \theta - \epsilon \). The second one, \( \epsilon \perp \epsilon \), is one-parametric class, where \( \epsilon_1 = \epsilon_2 = \epsilon \) and \( \theta = \pi/2 \). The third class consists of elliptically polarized standing waves (\( \epsilon_1 = \epsilon_2 = \epsilon \) and \( \theta = 0 \)).

4.1. The \( \epsilon \perp \epsilon \) Field Configuration

In this case, the scalar field invariants take the form

\[
\mathbf{E}^2 = 2 E_i^2 \left[ 1 - \sin(2\epsilon) \sin(2kz) \right];
\]

\[
\mathbf{J} = 2 E_i^2 \cos(2\epsilon) \cos(2kz).
\]

The gradients of the phase and of the angle \( \phi \) vanish. Consequently, the scattering force is zero for this configuration. If \( \epsilon \neq 0 \), both the intensity and ellipticity are spatially nonuniform. Equation (10) shows that extremal points of the intensity \( I \propto E^2 \) are correlated with points of circular polarization, where \( I = \cos(2\epsilon) \cos(2kz) \left[ 1 - \sin(2\epsilon) \sin(2kz) \right] = 0 \). In these points \( 2kz = \pi/2(2n + 1) \), the function \( \Psi \) is maximal in the case of \( J \rightarrow J + 1 \) transitions, and it has minima at these points for the \( J \rightarrow J \) transitions. The spatial dependence of the dipole force is shown in Fig. 2. One can observe double-well potential structures. These structures appear at positive detuning for the \( J \rightarrow J + 1 \) transitions near the intensity minima. In the case of \( J \rightarrow J \) transitions, double potential wells appear at negative detuning around the intensity maxima.
4.2. The ε – θ – Ė Field Configuration

Here, the field invariants read

\[ \mathcal{E}^2 = 2E_\perp^2[1 + \cos \theta \cos(2\varepsilon) \cos(2kz)]; \quad (11) \]
\[ \mathcal{F} = 4E_\perp^2[\cos \theta + \cos(2\varepsilon) \cos(2kz) + i \sin \theta \sin(2\varepsilon)], \]

and, generally speaking, all four gradients \( g_i \) (where \( i = 1; \ldots; 4 \)) are not equal to zero. The intensity extrema are correlated with points of the field linear polarization, where \( l = 1 \). At these points, the function \( \Psi \) is maximal in the case of \( J \rightarrow J \) transitions. For the \( J \rightarrow J + 1 \) transitions in the minima of \( I \), the function \( \Psi \) is also minimal. However, in the maxima of \( I \), the function \( \Psi \) can be either minimal or maximal depending on the angular momentum \( J \) and on the field parameters \( \theta \) and \( \varepsilon \). When the ellipticity angle is sufficiently small and the orientation angle is sufficiently close to \( \pi/2 \), double-well potential structures can appear (see Fig. 3). They are situated near the intensity maxima for the \( J \rightarrow J + 1 \) transitions at \( \delta < 0 \) and near the field minima in the case of \( J \rightarrow J \) transitions at \( \delta > 0 \). Since the phase and angle gradients differ from zero in the general case (\( \varepsilon \neq 0, \pm \pi/4 \) and \( \theta \neq 0 \)), the scattering force...
contributes to the optical potential. The scattering force vanishes in the points of linear polarization. At small detunings $|\delta| = \gamma$, the scattering force deforms the optical potential substantially.

5. NUMERICAL SIMULATIONS

We describe numerical simulations of the focusing in linear field configuration. The simulations are based on the semiclassical stochastic method introduced in the laser cooling theory [9]. The dipole force $F_0$ on an atom at rest for $J \rightarrow J + 1$ optical transitions for large $J$ has a good approximation to triangular spatial dependence. Thus, the area where the optical potential has a parabolic form increases for large $J$, and we expect a significant reduction of spherical aberration.

Apart from the force $F_0$, we take into account the frictional force and the random component corresponding to the momentum diffusion coefficient. It is worth noting that the relative influences of the dissipative and random forces on the atomic spatial distribution, compared to the main focusing force $F_0$, depend on the parameters of the problem (the light detuning and intensity, the interaction time, the initial divergence of the atomic beam, etc.), the light field configuration, and the type of atomic transition.

We study focusing for two different atomic transitions $1/2 \rightarrow 3/2$ and $4 \rightarrow 5$. The other parameters we choose according to the case of Cs atoms. In Fig. 4, the spatial dependencies of the optical potential $U(z)$ and the friction $\xi(z)$ and diffusion $D(z)$ coefficients are shown. The sharp features in $\xi(z)$ and $D(z)$ are seen in the case of large momenta ($4 \rightarrow 5$).

In our simulation we suppose that the initial beam momentum distribution has a Gaussian profile with $\Delta p_x = 10\hbar k$. The spatial distribution of atoms is shown in Fig. 5. One can see narrow subwavelength spatial structures (the FWHM widths are $\Delta x = 0.05\lambda$ in Fig. 5a and $\Delta x = 0.03\lambda$ in Fig. 5b) formed due to focusing in the thick lens regime. The interaction time is equal to a quarter of the oscillation period in the potential $U(z)$ ($t_{\text{int}} = 49.16\gamma^{-1}$ and $t_{\text{int}} = 41.37\gamma^{-1}$ in the cases of the $1/2 \rightarrow 3/2$ transition and the $4 \rightarrow 5$ transition, respectively). The difference in the spot size $\Delta x$ in these cases can be related to differences in the dissipative and random forces. As is seen from Fig. 5, the pedestal is significantly reduced in the case of $4 \rightarrow 5$ transitions (the ratio of the pedestal area to the total area $R = 0.19$) compared to the case of $1/2 \rightarrow 3/2$ transition ($R = 0.35$), which confirms our expectations concerning the reduced spherical aberrations.

We consider the time of the atomic beam interaction with light to be large in comparison with the optical pumping time $t \gg \Gamma_{\text{op}}^{-1}$. Nevertheless, two different interaction regimes for atom lithography can be distinguished: nondissipative ($t \ll \omega_{\text{rec}}^{-1}$) and dissipative ($t \geq \omega_{\text{rec}}^{-1}$) optical mask. We also numerically simulate the focusing for several 1D and 2D field configurations, taking into account dissipative forces and diffusion. In the 1D case, we point out the linear field configuration. The optical potential for atoms with optical transitions $J \rightarrow J + 1$ here become harmonics for a large area that leads to significant reduction of the spherical aberration of the optical mask in the nondissipative regime.

In the case of a long interaction time, we thoroughly analyze a number of 2D field configurations. Numerical simulations show that the spatial structure of these lattices is determined by the main focusing force $F_0$, and the influence of $F_{\text{diss}}$ and $F_{\text{rand}}$ reveals itself in modification of the shapes of localization sites.

REFERENCES